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Technical Notes

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Natural Convection Within a Vertical Finite-Length Channel in Free Space

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Introduction

NATURAL convection in a vertical semi-infinite channel has been extensively studied. Generally, the assumptions made by earlier researchers in their computational studies are 1) the inlet temperature and velocity profiles are uniform; 2) the only mode of heat transfer from the channel is natural convection; and 3) the channel length is assumed to be semi-infinite, i.e., the channel aspect ratio is large so that the boundary-layer approximations are valid. Most of the studies have dealt with boundary-layer flow; and the effect of the elliptic nature of the flow has not, to the authors' knowledge, been definitively established.

Recently, Ramanathan and Kumar[1] presented the numerical results of natural convective flows between two vertical parallel finite-length plates within a large solid enclosure. Their results were in good agreement with those of the semi-infinite channel cases reported in the literature for large aspect ratios, but not for small aspect ratios. They concluded that this discrepancy was due to the neglect of thermal diffusion in the vertical direction of the channel. They also addressed that the geometric dimensions of the large solid enclosure was maintained at appropriate values such that the enclosure had no effect on the results. According to our observation, however, heat transfer in a finite-length channel is influenced by the heat transfer between the fluid within the solid enclosure and the enclosure walls. Furthermore, their definition of Nusselt number based on the centerline temperature at the channel entrance, which varied under different Rayleigh numbers and aspect ratios, was not appropriate for interpreting the heat transfer characteristics of the finite-length channel in free space, especially for low Rayleigh numbers and small aspect ratios. In addition, no information related to induced Reynolds number in such a configuration was reported in their study. For this reason, the natural convection within a vertical finite-length isoflux channel in free space is systematically examined in the present study. The objectives of this study are to remove any assumptions that need to be made on the velocity and temperature profiles at the channel entrance; to include the effects of vertical thermal diffusion and free space stratification; to compare the present results with those presented in the existing literature; and to propose a new correlation of induced Reynolds number in a vertical finite-length channel.

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Theoretical Analysis

A vertical finite-length channel in free space is considered in the present study. Two-dimensional, incompressible laminar natural convection is induced by the buoyancy force in the fluid. Fluid properties are assumed constant; the viscous dissipation and the compressibility effect in the energy equation are neglected.

Let

$$X = \frac{x}{H}; \quad U = \frac{uH}{\nu}; \quad \tau = \frac{t\nu}{H^2}; \quad P = \frac{(p - p_0)H^2}{\rho\nu^2}$$

$$\Theta = \frac{T - T_0}{qH/k}; \quad Pr = \frac{\nu}{\alpha}; \quad Y = \frac{y}{H}; \quad V = \frac{vH}{\nu} \tag{1}$$

$$A = \frac{L}{H}; \quad \beta = \frac{1}{\rho}\left(\frac{\rho - \rho_0}{T - T_0}\right); \quad Gr = \frac{\beta g q H^4}{k\nu^2}$$

where T_0, p_0 and ρ_0 represent the ambient temperature, pressure, and density at the location far away from the channel in free space, respectively; x and y are the coordinates parallel to and normal to the channel surface, respectively; H is the channel spacing; and q is the constant heat flux dissipated from the surfaces inside the channel.

With the above-mentioned assumptions and the Boussinesq approximation, the governing equations can be written in dimensionless form as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2}$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Gr\Theta + \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \tag{3}$$

$$\frac{\partial V}{\partial \tau} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \tag{4}$$

$$\frac{\partial \Theta}{\partial \tau} + U\frac{\partial \Theta}{\partial X} + V\frac{\partial \Theta}{\partial Y} = \frac{1}{Pr}\left(\frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2}\right) \tag{5}$$

Since the present computational domain is symmetric, only half of the configuration is needed to simulate the problem. Moreover, the channel-plate thickness is assumed much smaller than the channel spacing, say $H/25$, the conduction heat transfer within the plate itself can be neglected. Thus, the dimensionless boundary conditions over all boundaries of the domain can be expressed as

1) On the centerline of channel:

$$\frac{\partial U}{\partial Y} = \frac{\partial V}{\partial Y} = \frac{\partial P}{\partial Y} = \frac{\partial \Theta}{\partial Y} = 0 \quad \text{at} \quad Y = 0 \tag{6}$$

2) On the channel planes:

$$U = V = 0 \quad \frac{\partial\Theta}{\partial Y} = 1 \quad \text{at} \quad 0 < X < A,\ Y = 0.5$$
$$U = V = 0 \quad \frac{\partial\Theta}{\partial Y} = 0 \quad \text{at} \quad 0 < X < A,\ Y = 0.54 \tag{7}$$

3) Boundary conditions in free space:

$$U = V = P = \Theta = 0 \qquad \text{at} \qquad X \to \pm\infty,\ Y \to \infty \tag{8}$$

For selecting a reasonable computational domain of $-L_1/H \leq X \leq A + L_3/H$ and $0 \leq Y \leq 0.54 + L_2/H$ in numerical calculations, the boundary conditions of Eq. (8) become

$$U = V = P = \Theta = 0$$
$$\text{at} \quad X = -L_1/H,\ X = A + L_3/H$$
$$\text{and} \quad Y = 0.54 + L_2/H \tag{9}$$

The above governing equations can be integrated over the control volume and then discretized by using the implicit backward difference method and power-law scheme with staggered grids. A time-marching procedure with a semi-implicit iterative algorithm, SIMPLEC, presented by van Doormaal and Raithby[2] is implemented for steady-state solutions in the present study.

Results and Discussion

The parameters studied are Prandtl number Pr 0.7, Rayleigh numbers Ra between 1.4 and 35,000, and channel aspect ratios A ranging from 1 to 5 in the present study.

To achieve an acceptable numerical accuracy and to save CPU time, five sizes of computational domain and four grid sizes were tested. From a comparison of these results, the computational domain $L_1 = L_2 = 20L$, $L_3 = 25L$ and grid size 82 × 68 were chosen for providing numerical results of high accuracy.

Heat Transfer Characteristics

A composite Nu correlation for natural convection in a vertical long channel with asymmetrical isoflux heating was proposed by Hung et al.[3] This proposed correlation was identical to that presented by Bar-Cohen and Rohsenow[4] for channel plates with symmetric isoflux heating. The correlation for symmetric isoflux heating was expressed as

$$Nu_0 = \left(\frac{12}{Ra} + \frac{1.877}{Ra^{0.4}}\right)^{-0.5} \tag{10}$$

where Nu_0 is defined in terms of the midheight (or approximately average) wall temperature and channel spacing H, and Ra is the channel Rayleigh number defined as $GrPr/A$.

For the cases of a finite-length channel in free space, Fig. 1 displays the Nusselt numbers at the midheight of the channel for various aspect ratios and Rayleigh numbers. A significant deviation between the present results and the predictions by Eq. (10) can be found for cases with small channel aspect ratios and low Rayleigh numbers. The reason is that the vertical heat conduction in the channel and the free space stratification near the channel are significant when both aspect ratio and Rayleigh number are small. The present Nusselt number at the midheight of the channel is higher than that predicted by a long-channel correlation. For comparison, the theoretical results of Ramanathan and Kumar[1] are also shown in the figure. A significant deviation of their data from the present results can be found for the case of $A = 1$. This deviation results from a different definition of Nusselt number they used in their study. Their Nusselt number (Nu_a) was based on a variable temperature reference, i.e., the centerline

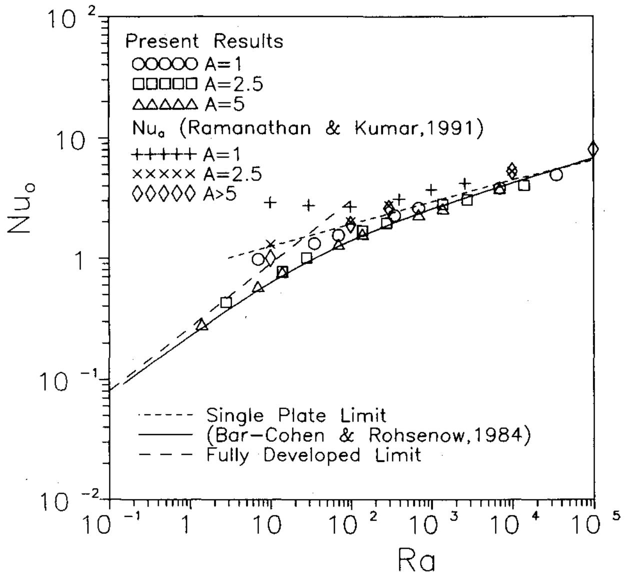

Fig. 1 Relationship between average Nusselt number and Rayleigh number.

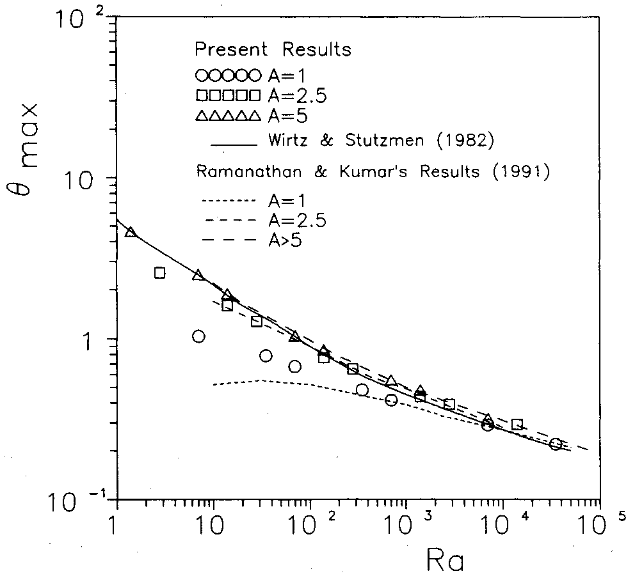

Fig. 2 Relationship between maximum dimensionless temperature and Rayleigh number.

temperature at the channel entrance. When the aspect ratio is large, i.e., $A \geq 2.5$, this centerline temperature approaches the ambient temperature T_0 in free space, while it will be affected by the vertical thermal diffusion and free space stratification when the aspect ratio becomes small, say $A = 1$. The centerline temperature at the channel entrance is usually higher than T_0 in free space, and consequently their average Nusselt numbers are higher than the present results.

Maximum Temperature on the Channel Surface

Figure 2 shows the effects of channel Rayleigh number and aspect ratio on the maximum dimensionless temperature of the channel surface. The present results for large aspect ratio, say $A = 5$, are in agreement with the experimental data for $A \gtrsim 17$ presented by Wirtz and Stutzman.[5] However, for $A < 5$ (e.g., $A = 2.5$ or 1) and low Rayleigh numbers, the present results deviate significantly from the theoretical and experimental data for $A \gtrsim 5$. This verifies that vertical heat conduction in the channel and free space stratification near the channel have a significant effect on the maximum temperature distribution for small channel aspect ratios and Rayleigh numbers. As shown in the figure, for $A = 2.5$ and 5, the present results are in good agreement with those reported by Ramanathan and Kumar. However, a significant deviation can be found for the case of $A = 1$. The main reason is due to the influence of heat transfer between the fluid inside the solid enclosure and the enclosure wall, resulting in a lower maximum dimensionless temperature on the channel surface in their results.

Induced Reynolds Number

A generalized derivation of the induced mass flow rate for the fully developed limit of natural convection in a vertical long channel with asymmetric isoflux heating was presented by Hung et al.[3] Accordingly, the fully developed limit of modified induced Reynolds number in a vertical channel with symmetrical isoflux heating can be expressed as

$$Re^* = \sqrt{\tfrac{1}{12}}Ra^{-0.5} \tag{11}$$

where Re^* is defined as $Re^* = Q/Gr$, with Q denoting the dimensionless induced flow rate in the channel, i.e.

$$Q = \int_0^1 U \, dY$$

and Gr as defined in Eq. (1).

From the scale analysis, the modified induced Reynolds number of a vertical long plate was found to be inversely proportional to the 0.6 power of Rayleigh number.[6] By checking with the present results, the single plate limit of the modified induced Reynolds number can be quantitatively expressed as

$$Re^* = 0.5Ra^{-0.6} \tag{12}$$

Similar to the method suggested by Churchill and Usagi[7] for correlating a composite Nusselt number, a new composite Re^* correlation for a vertical channel having arbitrary channel spacing is proposed as

$$Re^* = [(\sqrt{\tfrac{1}{12}}Ra^{-0.5})^{-6} + (0.5Ra^{-0.6})^{-6}]^{-1/6} \tag{13}$$

As indicated in Fig. 3, the data from the present study are very consistent with the results of the boundary-layer analysis presented by Miyatake and Fujii,[8] except for $A = 1$. This shows that the vertical heat conduction in the channel and free space stratification near the channel have a significant effect on the induced Reynolds number when the channel aspect ratios are small.

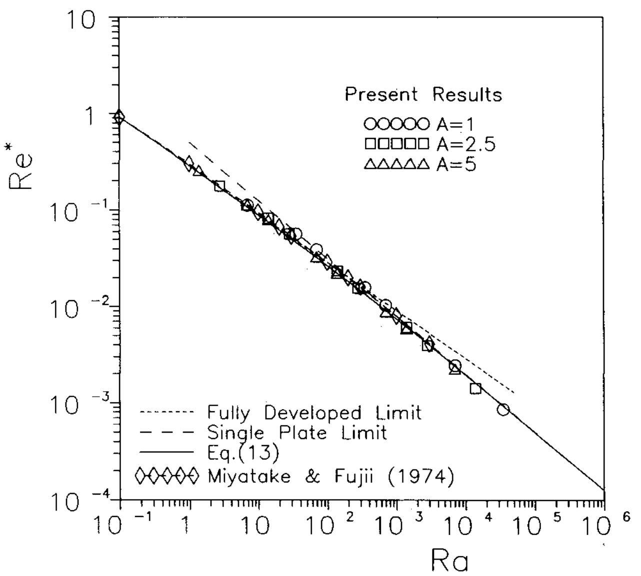

Fig. 3 Relationship between induced Reynolds number and Rayleigh number.

Conclusions

The main conclusions from the present results can be summarized as follows:

1) For small channel aspect ratios and low Rayleigh numbers, significant deviations of the Nusselt number and maximum temperature distributions exist between the present results and the predications from the existing long-channel correlations. This discrepancy is due to the fact that the effects of vertical thermal diffusion in the channel and free space stratification near the channel are significant.

2) A new correlation of the induced Reynolds number for a vertical finite-length isoflux channel is proposed.

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Prediction of Radiative Transfer from Potential Core of a Hot Jet

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Nomenclature

a_λ = absorption coefficient of species as a function of wavelength
i'_λ, $i'_{\lambda b}$ = intensity and blackbody intensity at a point in medium as a function of direction and wavelength
R = radius of nozzle face

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